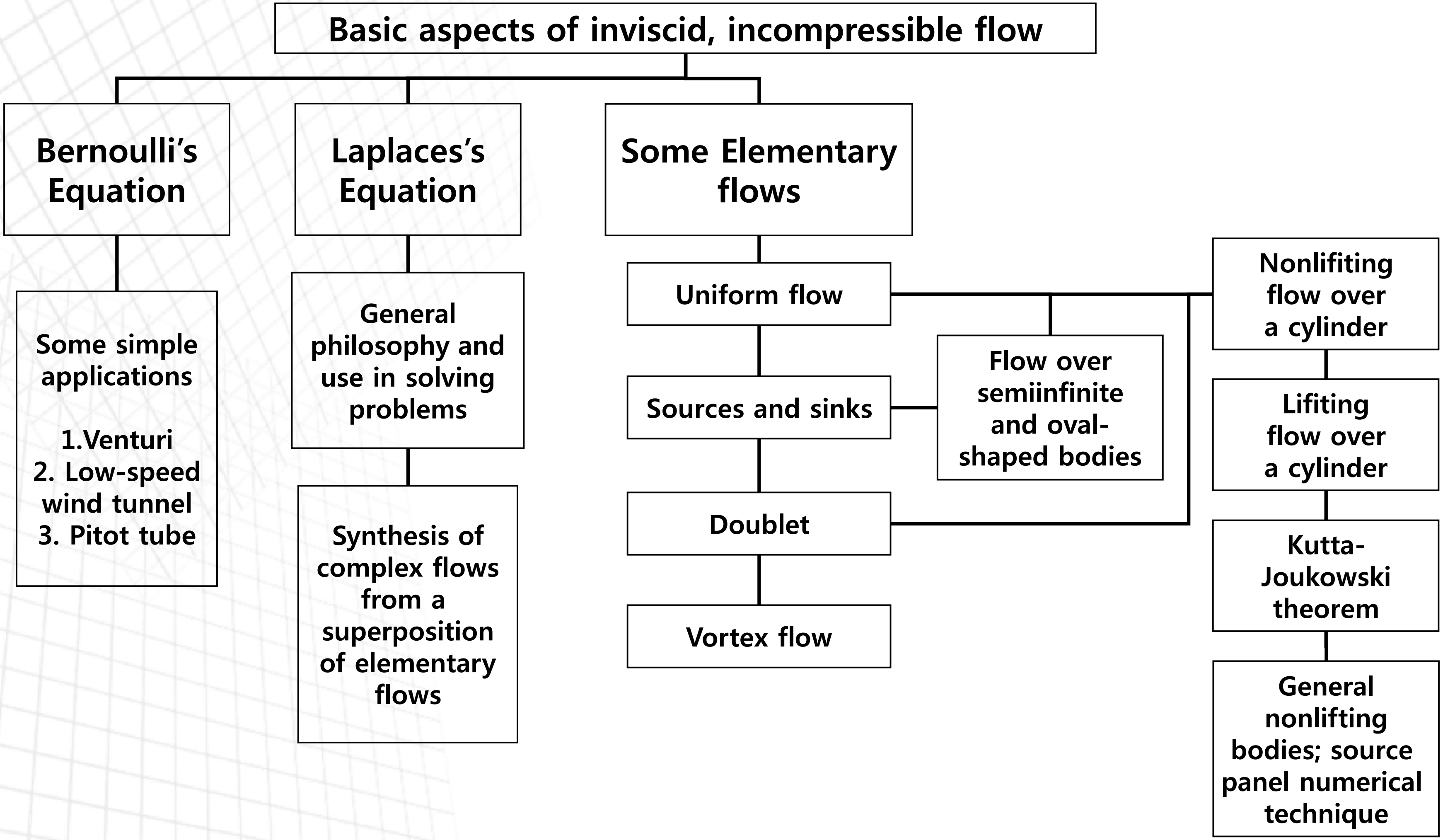
< 3.1. Introduction and Road Map >



< 3.2. Bernoulli's Equation >

❖ **From the momentum equation**

$$\rho\frac{\partial \vec{V}}{\partial t}+(\vec{V}\bullet\nabla)\rho\vec{V}=-\nabla p+\rho\vec{f}+F_{viscous}$$

❖ **For an inviscid and steady flow without no body force**

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\mathrm{w}\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}$$: x-dir. Momentum equation

< 3.2. Bernoulli's Equation >

- **Multiply dx**

$$u\frac{\partial u}{\partial x}dx + v\frac{\partial u}{\partial y}dx + w\frac{\partial u}{\partial z}dx = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

- **From the equation of streamline**

$$udz - wdx = 0$$

$$vdx - udy = 0$$

➔ $$u\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

< 3.2. Bernoulli's Equation >

$$u\left(\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy+\frac{\partial u}{\partial z}dz\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

➔ $$udu=-\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

➔ $$\frac{1}{2}d(u^2)=-\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$: x-dir.

Similarly, $$\frac{1}{2}d(v^2)=-\frac{1}{\rho}\frac{\partial p}{\partial y}dy$$: y-dir.

$$\frac{1}{2}d(w^2)=-\frac{1}{\rho}\frac{\partial p}{\partial z}dz$$: z-dir.

< 3.2. Bernoulli's Equation >

➔ $\frac{1}{2}d(u^2+v^2+w^2)=-\frac{1}{\rho}dp$

➔ $dp=-\rho V dV$ along a streamline

If incompressible, $\int_1^2 dp=-\rho\int_1^2 V dV$ ➔ $p_2-p_1=-\rho(\frac{V_2^2}{2}-\frac{V_1^2}{2})$

$\therefore p_1+\frac{1}{2}\rho V_1^2=p_2+\frac{1}{2}\rho V_2^2$ ➔ $p+\frac{1}{2}\rho V^2=const$ along a streamline

If irrotational, $p+\frac{1}{2}\rho V^2=const$ everywhere

< 3.3. The Venturi and Low-Speed Wind Tunnel >

❖ Venturi tube

- Assume)
  - 1. Quasi-one-dimensional flow
    (the properties are uniform across the x-section)
  - 2. Inviscid flow
  - 3. Steady flow
- Continuity equation

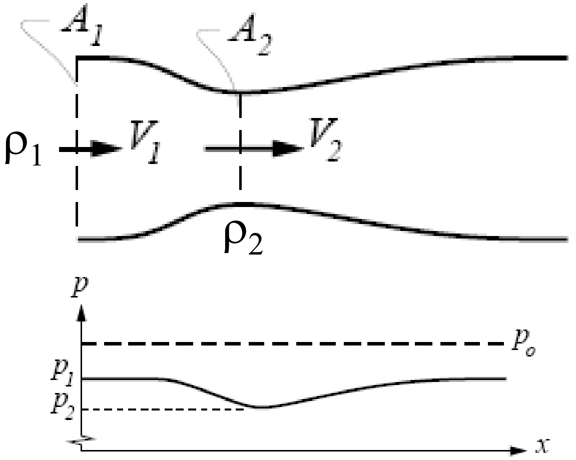
$$\frac{\partial}{\partial t}\iiint_V \rho \, dV + \oiint_S \rho \vec{V} \cdot d\vec{S} = 0$$

steady

$$\rightarrow \iint_{A_1 + A_2 + wall} \rho \vec{V} \cdot d\vec{S} = 0$$

< 3.3. The Venturi and Low-Speed Wind Tunnel >

❖ Venturi tube



$$\iint_{A_1 + A_2 + wall} \rho \vec{V} \cdot d\vec{S} = 0$$

$$\iint \rho \vec{V} \cdot d\vec{S} = 0$$ at the wall

$$1 : \iint_{A_1} \rho \vec{V} \cdot d\vec{S} = -\rho_1 A_1 V_1$$

$$2 : \iint_{A_2} \rho \vec{V} \cdot d\vec{S} = \rho_2 A_2 V_2$$

$$\therefore \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

< 3.3. The Venturi and Low-Speed Wind Tunnel >

❖ Venturi tube

Incompressible ➔ $A_1 V_1 = A_2 V_2$

Use two equations

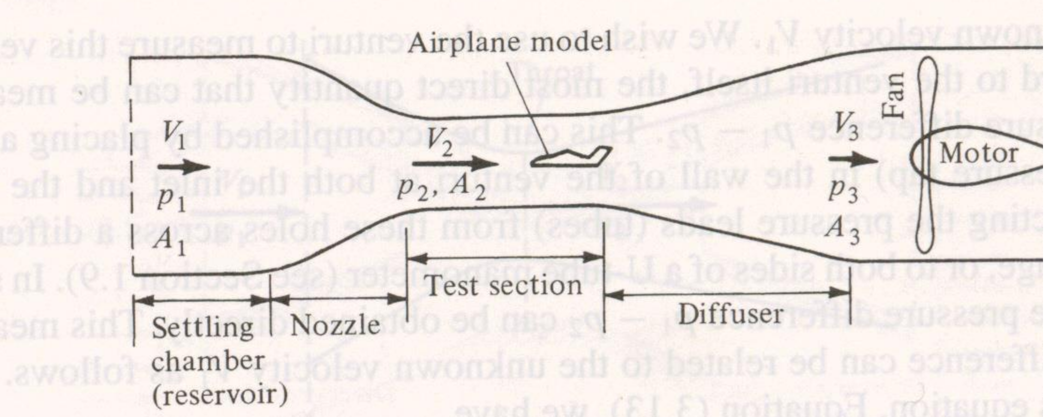
1. Continuity eq. $AV = const$
2. Bernoulli's eq. $p + \frac{1}{2}\rho V^2 = const$

➔ $V_1^2 = \frac{2}{\rho}(p_2 - p_1) + V_2^2 = \frac{2}{\rho}(p_2 - p_1) + (\frac{A_1}{A_2} V_1)^2$

$$\therefore \; V_1 = \sqrt{\frac{2(p_2 - p_2)}{\rho[(\frac{A_1}{A_2})^2 - 1]}}$$

< 3.3. The Venturi and Low-Speed Wind Tunnel >

❖ Low-speed wind tunnel



(a) Open-circuit tunnel

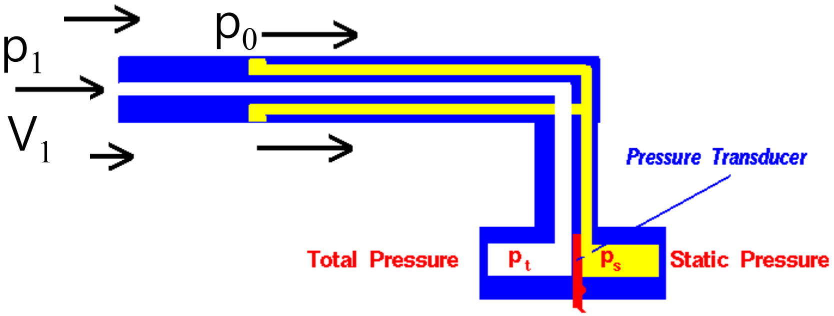
$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1-(A_2/A_1)^2]}}$$

How to measure
the pressure difference?

→ Manometer

< 3.4. Pitot Tube >

❖ Low-speed wind tunnel



$$V_1 = \sqrt{\frac{2(p_0 - p_1)}{\rho}}$$

$$\frac{1}{2}\rho V_1^2 + p_1 = p_0$$

Dynamic pressure → $\frac{1}{2}\rho V_1^2$

Static pressure → p_1

Total pressure → p_0

< 3.5. Pressure Coefficient >

❖ Low-speed wind tunnel

$$C_p \equiv \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}$$: Over all the range of speed

For incompressible flow

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p + \frac{1}{2}\rho V^2 \;\Rightarrow\; p - p_\infty = \frac{1}{2}\rho(V_\infty^2 - V^2)$$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{\frac{1}{2}\rho(V_\infty^2 - V^2)}{\frac{1}{2}\rho V_\infty^2} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

$$\therefore\; C_p = 1 - \left(\frac{V}{V_\infty}\right)^2$$: works for M<0.3 (low subsonic)

* freestream ➔ $C_p = 0$ * stagnation point ➔ $C_p = 1$

< 3.6. Condition on Velocity for Incompressible Flow >

❖ Continuity equation

Incompressible ➔ $$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{V}) = 0$$

➔ $$\therefore \nabla \bullet \vec{V} = 0$$

< 3.7. Laplace's Equation >

$$\nabla \cdot \vec{V} = 0$$
$$\vec{V} = \nabla \phi$$

➔ $\nabla \cdot (\nabla \phi) = \nabla^2 \phi = 0$ ➔ Laplace's equation

* In 2D,

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

* In irrotational flow,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$
$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = 0$$
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

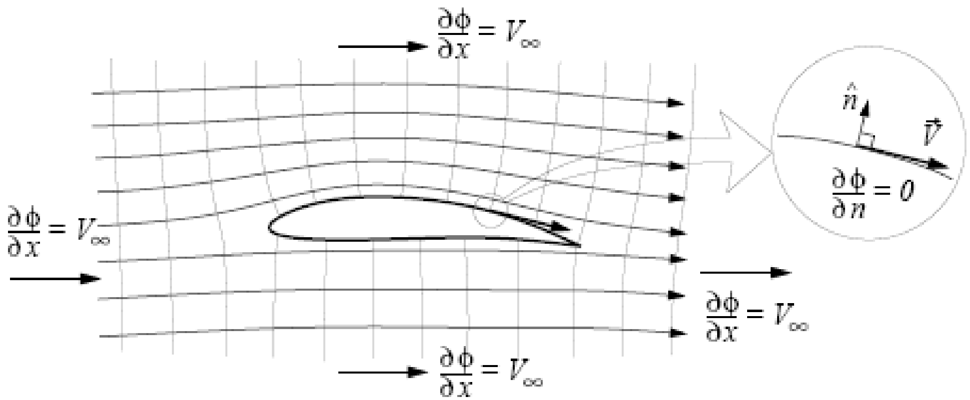
Note)
1. For irrotational, incompressible flow, and are both solutions of Laplace equation.
2. Since Laplace equation is linear, the solution can be superimposed, so that any complex flow is expressed by adding elementary.

< 3.7. Laplace's Equation >

i) Infinity B.C.

$$u = \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = V_\infty$$

$$v = \frac{\partial \psi}{\partial y} = -\frac{\partial \psi}{\partial x} = 0$$



ii) wall B.C.

$$\left(\frac{v}{u}\right)_{wall} = \left(\frac{dy_b}{dx}\right)$$ ← Flow tangency condition

slope of the streamline